

Practice Problems *for the* Mechanical Engineering PE Exam

**A Companion to the
*Mechanical Engineering Reference Manual***

Twelfth Edition

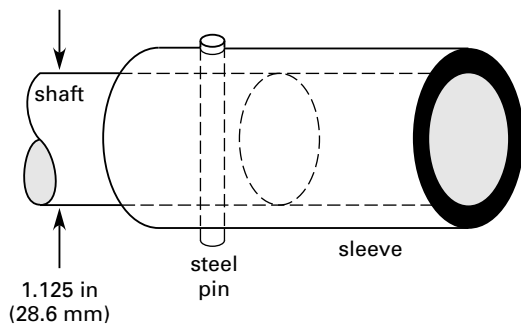
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Failure Theories

PRACTICE PROBLEMS

1. A shaft with a 1.125 in (28.6 mm) diameter receives 400 in-lbf (45 N·m) of torque through a pinned sleeve. The pin is manufactured from steel with a tensile yield strength of 73.9 ksi (510 MPa). Using a factor of safety of 2.5, what pin diameter is needed?

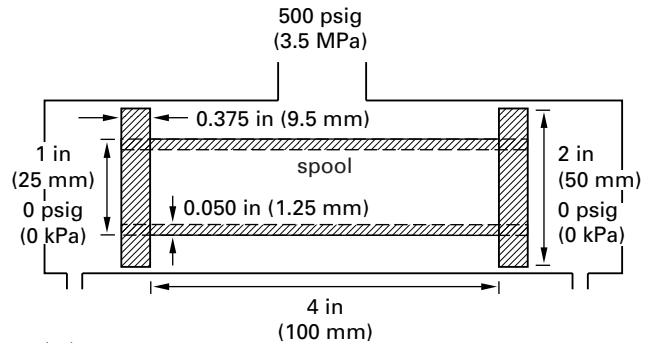


- (A) 0.16 in (0.0041 m)
- (B) 0.19 in (0.0048 m)
- (C) 0.26 in (0.0066 m)
- (D) 0.37 in (0.0094 m)

2. A $\frac{5}{8}$ in UNC bolt (with rolled threads) and nut carry a simple tensile load that fluctuates between 1000 lbf and 8000 lbf (4400 N and 35,000 N). Both the bolt and nut are made from a material with a yield strength of 70,000 lbf/in² (480 MPa) and an endurance limit of 30,000 lbf/in² (205 MPa). Including a stress concentration factor for the threads, what is the factor of safety for the bolt?

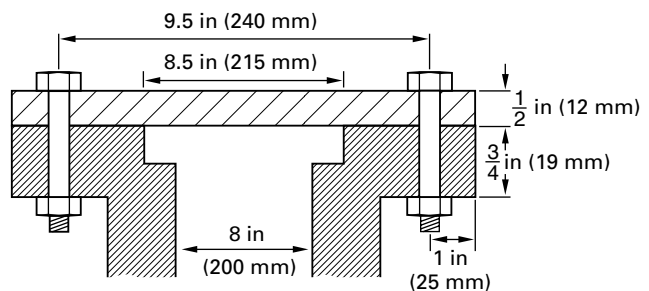
- (A) 0.5
- (B) 0.7
- (C) 1.2
- (D) 2.4

3. (*Time limit: one hour*) The spool in the spool valve shown is constructed of aluminum with a yield strength of 19,000 lbf/in² (130 MPa). A 500 psi (3.5 MPa) pressure differential exists across the spool shaft. The shaft has a 1.0 in (25 mm) outside diameter and a 0.050 in (1.3 mm) wall thickness. The end disks have a 2.0 in (50 mm) outside diameter and a 0.375 in (9.5 mm) thickness. The end disks are rigidly attached to the tube. Disregard spool collapse. Use the distortion energy theory to calculate the factor of safety for the spool.



- (A) 0.8
- (B) 1.1
- (C) 1.6
- (D) 2.3

4. (*Time limit: one hour*) The pressure in a small pressure vessel varies continually between the extremes of 50 psig and 350 psi (340 kPa and 2400 kPa). The vessel is closed by a $\frac{1}{2}$ in (12 mm) plate. The plate is attached to the vessel flange with six $\frac{3}{8}$ -24 UNF bolts evenly spaced around a $9\frac{1}{2}$ in (240 mm) circle. Each bolt is tightened to an initial preload of 3700 lbf (16.4 kN). The bolts and nuts are constructed of cold-rolled steel with a 90 ksi (620 MPa) yield strength and 110 ksi (760 MPa) ultimate strength. The plate, flange, and vessel are constructed of steel with a 30 ksi (205 MPa) yield strength and a 50 ksi (345 MPa) ultimate strength. The vessel is intended to be used indefinitely. Neglect the effects of the gasket (not shown). Neglect bending of the plate. Using a thread stress concentration factor of 2 and appropriate endurance strength derating factors, what is the factor of safety?



(sealing method not shown)

- (A) 1.2
- (B) 1.8
- (C) 2.1
- (D) 2.5

SOLUTIONS

1. Customary U.S. Solution

Use the distortion energy failure theory. From Eq. 50.27, the yield strength in shear is

$$S_{ys} = 0.577S_{yt} = (0.577) \left(73.9 \times 10^3 \frac{\text{lbf}}{\text{in}^2} \right) = 42.64 \times 10^3 \text{ lbf/in}^2$$

For a safety factor of 2.5, the maximum allowable shear stress is

$$\tau_{\max} = \frac{S_{ys}}{\text{FS}} = \frac{42.64 \times 10^3 \frac{\text{lbf}}{\text{in}^2}}{2.5} = 17.06 \times 10^3 \text{ lbf/in}^2$$

The total shear at the pin is

$$V = \frac{\text{shaft torque}}{\text{shaft radius}} = \frac{400 \text{ in}\cdot\text{lbf}}{\frac{1.125 \text{ in}}{2}} = 711.1 \text{ lbf}$$

This is not a case of biaxial stress. The direct shear stress in the pin is

$$\tau_{\max} = \frac{V}{A}$$

Solve for the required total pin area.

$$A = \frac{V}{\tau_{\max}} = \frac{711.1 \text{ lbf}}{17.06 \times 10^3 \frac{\text{lbf}}{\text{in}^2}} = 0.04168 \text{ in}^2$$

Since two surfaces of the pin resist the shear,

$$A = (2) \left(\frac{\pi d^2}{4} \right) = \frac{\pi d^2}{2}$$

Solve for the pin diameter.

$$d = \sqrt{\frac{2A}{\pi}} = \sqrt{\frac{(2)(0.04168 \text{ in}^2)}{\pi}} = 0.163 \text{ in}$$

The answer is (A).

SI Solution

Use the distortion energy failure theory. From Eq. 50.27, the yield strength in shear is

$$S_{ys} = 0.577S_{yt} = (0.577) (510 \times 10^6 \text{ Pa}) = 294.3 \times 10^6 \text{ Pa}$$

For a safety factor of 2.5, the maximum allowable shear stress is

$$\tau_{\max} = \frac{S_{ys}}{\text{FS}} = \frac{294.3 \times 10^6 \text{ Pa}}{2.5} = 117.7 \times 10^6 \text{ Pa}$$

The total shear at the pin is

$$V = \frac{\text{shaft torque}}{\text{shaft radius}} = \frac{45 \text{ N}\cdot\text{m}}{\frac{0.0286 \text{ m}}{2}} = 3146.9 \text{ N}$$

This is not a case of biaxial stress. The direct shear stress in the pin is

$$\tau_{\max} = \frac{V}{A}$$

Solve for the required total pin area.

$$A = \frac{V}{\tau_{\max}} = \frac{3146.9 \text{ N}}{117.7 \times 10^6 \text{ Pa}} = 2.674 \times 10^{-5} \text{ m}^2$$

Since two surfaces of the pin resist the shear, the total pin area is

$$A = (2) \left(\frac{\pi d^2}{4} \right) = \frac{\pi d^2}{2}$$

Solve for the pin diameter.

$$d = \sqrt{\frac{2A}{\pi}} = \sqrt{\frac{(2)(2.674 \times 10^{-5} \text{ m}^2)}{\pi}} = 0.00413 \text{ m}$$

The answer is (A).

2. Customary U.S. Solution

From Table 51.5, the tensile stress area for a ⁵/₈ in UNC bolt is $A = 0.226 \text{ in}^2$.

The maximum stress is

$$\sigma_{\max} = \frac{F_{\max}}{A} = \frac{8000 \text{ lbf}}{0.226 \text{ in}^2} = 35,398 \text{ lbf/in}^2$$

The minimum stress is

$$\sigma_{\min} = \frac{F_{\min}}{A} = \frac{1000 \text{ lbf}}{0.226 \text{ in}^2} = 4425 \text{ lbf/in}^2$$

From Eq. 50.28, the mean stress is

$$\begin{aligned} \sigma_m &= \frac{\sigma_{\max} + \sigma_{\min}}{2} = \frac{35,398 \frac{\text{lbf}}{\text{in}^2} + 4425 \frac{\text{lbf}}{\text{in}^2}}{2} \\ &= 19,912 \text{ lbf/in}^2 \end{aligned}$$

From Eq. 50.30, the alternating stress is

$$\begin{aligned} \sigma_{\text{alt}} &= \frac{1}{2}(\sigma_{\max} - \sigma_{\min}) \\ &= \frac{1}{2} \left(35,398 \frac{\text{lbf}}{\text{in}^2} - 4425 \frac{\text{lbf}}{\text{in}^2} \right) \\ &= 15,487 \text{ lbf/in}^2 \end{aligned}$$

From Sec. 51.11, the stress concentration factor for rolled threads is 2.2. Thus, the alternating stress is

$$\sigma_{\text{alt}} = \left(15,487 \frac{\text{lbf}}{\text{in}^2} \right) (2.2) = 34,071 \text{ lbf/in}^2$$